Roll No. ....

72452

M.Sc. Mathematics 1st Semester Examination–December, 2014

REAL ANALYSIS-I

Paper : MM-412

Time : 3 hours ***Max. Marks : 80***

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard will be entertained after the examination.

Note : Attempt **five** questions in all selecting atleast **one** question from each Section. Questions from Section-V are **compulsory**.

SECTION – I

1. (a) Let α be monotonic increasing function on [a, b]. Then prove that $f \in R(\alpha)$ if an only if for each $\in > o$, there exists a partition P of [a, b] such that

$$U(P,f,\alpha) - L(P, f, \alpha) < \in$$

(b) Suppose $f \in R(\alpha)$ on [a,b] and $m \le f \le M$. ϕ is continuious function on [m,M] and h(x)

$= \phi(f(x)$ on $[a,b]$ then prove that $h \in R(\alpha)$ on $[a,b]$.

2. (a) Evaluate

(i) $\int_0^2 [x]\, dx^2$

(ii) $\int_0^3 x\, d([x] - x)$

(b) If f maps $[a, b]$ into R^k and if $f \in R(\alpha)$ for some monotonically increasing function α an $[a, b]$. Then prove that $|f| \in R|\alpha$ and $\left|\int_a^b f d\alpha\right| \leq \int_a^b |f| d\alpha$.

SECTION – II

3. (a) Let $\{A_n\}$ be a countable collection of sets of real numbers then. Show that

$$m^*\left[\bigcup_n A_n\right] \leq \sum_n m^*(A_n)$$

(b) Prove that outer measure is translation invariant.

4. (a) Prove that a countable union of measurable sets is a measurable set.

(b) Let E be a set with outer measure finite. Then prove that E is measurable if and only if given $\epsilon > o$, there is a finite union B of open intervals such that $m^*(E \Delta B) < \epsilon$.

SECTION – III

5. (a) Let f be a continuous function and g be a measurable function. That prove that fog is measurable.

(b) Prove that the set of points on which a sequence of measurable functions converges is measurable.

6. (a) If f is a measurable function and f = ga.e in E. Then show then g is measurable.

(b) Let $f : [a, b] \to R$ be a measurable function, then show that given $\phi > o$, that exists a closed subset F of E=[a,b] such that m (E-F) < ϕ and f/F is continuous.

SECTION – IV

7. (a) Let f be a bounded function on [a, b]. If f is Riemann inrtegrable on [a, b] then prove that f is measurable and

$$R \int_a^b f(x)\,dx = \int_a^b f(x)\,dx$$

(b) State and prove Lebesgue Bounded convergence Theorem.

8. (a) If f and g are non negative measurable functions then prove that :

$$\int_E (f + g) = \int_E f + \int_E g$$

(b) Show that the monotone convergence theorem need not good for decreasing sequence of functions.

(c) If f is integrable function. Then show that f is finite valued a.e.

SECTION – V

9. (a) Define norm of a partition.

(b) Define Riemann-Stieltjes integral as a limit of sum

(c) State Heine Borel theorem

(d) Define Boolean algebra

(e) Define characteristic function of a set

(f) State littlewood's third principle of measurability

(g) State Fatou's Lemma

(h) State Lebesgue Dominated Convergence Theorem